# About the strong EULER-GOLDBACH conjecture

PHILIPPE SAINTY

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PHILIPPE SAINTY
University Pierre et Marie Curie Paris, France.
July 2022

Abstract.
In this article, we define a " recursive local" algorithm in order to construct two reccurent numerical sequences of positive prime numbers $(U_{2n})$ and $(V_{2n})$, ($(U_{2n})$ function of $(V_{2n})$), such that for any integer n≥ 2, their sum is 2n. To build these , we use a third sequence of prime numbers $(W_{2n})$ defined for any integer n≥ 3 by : $W_{2n}$ = Sup(p∈IP : p ≤ 2n-3), where IP is the infinite set of positive prime numbers. The Goldbach conjecture has been verified for all even integers 2n between 4 and $4.10^{18}$. In the Table of Goldbach sequence terms given in paragraph § 10, we reach values of the order of 2n= $10^{1000}$. Thus, thanks to this algorithm of "ascent and descent", we can validate the strong Euler-Goldbach conjecture.

Keywords.
Prime numbers, Prime Number Theorem, weak and strong Goldbach's conjectures, Dirichlet's theorem, Bertrand-Tchebychev theorem, gaps between prime numbers.

## CONTENTS.



## § 1 Preliminaries and history.

Number theory, "the Queen of Mathematics" deals with structures and properties defined over integers (see Euclide [5], Hardy & Wright [7 ] and Landau [10]). This theory is built from the notion of prime number. Many problems have been raised and multiple conjectures with statements that are often simple but very difficult to demonstrate have been made. These main components include:

a) Elementary arithmetic.
Determination and properties of prime numbers.

Operations on integers (basic operations, congruence, gcd, lcm, and so on…….).

Representations and numbers bases of integers.

Decomposition of integers into product or sum of prime numbers (Fundamental theorem of arithmetic, decomposition of large numbers, cryptography and Goldbach's conjecture).

b) Analytic number theory.

(Riemann Hypothesis). Distribution of prime numbers (Prime Number Theorem, see Hadamard [6], De la Vallée-Poussin [18], Littlewood [11] and Erdos [4]), gaps between prime numbers (see Bombieri [1], Cramer [3], Iwaniec & Pintz [9] and Tchebychev [17]).

c) Algebraic, probabilistic, combinatorics and algorithmic theories of numbers.

d) Modular arithmetic, diophantine approximations, equations and geometry. Arithmetic sequences, functions, and algebraic geometry.

As part of research on the Goldbach conjecture, Chen [2], Hegfollt [8], Ramaré [12], Tchebychev [17], Vinogradov [19], Tao [16] and Zhang [20] have taken essential steps and obtained very promising results for its demonstration. In effect, Helfgott & Platt [8] proved in 2013 Goldbach's weak conjecture. Silva, Herzog & Pardi [15] held the record for calculating the terms of Goldbach sequences after having determined, pairs of positive prime numbers $(p_n;q_n)$ verifying : $(p_n + q_n = 2n)$ for any integer n such that: $(4 \le 2n \le 4.10^{18})$. In the scientific literature, one does not find explicit constructions of reccurent sequences of positive Goldbach primes $(G_{2n}) = (U_{2n};V_{2n})$, satisfying for any integer n ≥ 2 the equality: $(U_{2n} + V_{2n} = 2n)$. In this article, we define two sequences of prime numbers, using a "local recursive" algorithm simple and efficient allowing to calculate for any integer n≥ 3, by successive iterations any term $U_{2n}$ and $V_{2n}$ of a Goldbach sequence. Using the scientific calculation software Maxima on a personal computer, one easily exceed Silva's record [15] set for $2n = 4.10^{18}$ and we can reach $2n = 10^{500}$ and even $2n = 10^{1000}$ .

§ 2 Definitions and notations.

a) The integers n, k, p, q, r,…..and so on…. are always positive.
b) We denote by IP the infinite set of positive prime numbers (called simply prime numbers):

IP= { $p_k$, (k ∈ IN*) : $p_k$ is the kth-positive prime number; ($p_k < p_{k+1}$ and lim $p_n$=+∞}

( $p_1$=2 ; $p_2$=3 ; $p_3$=5 ; $p_4$=7 ; p5=11 ; $p_6$=13; ……….).

To simplify the writing of large numbers, we adopt the following notations .

c) $M = 10^9$
d) $R = 4.10^{18}$
e) $G = 10^{100}$
f) $S = 10^{500}$
g) $T = 10^{1000}$
h) log(x) denotes the decimal logarithm of the strictly positive real x : (x> 0).
i) We call Goldbach sequence any numerical sequence denoted by $(G_{2n}) = (U_{2n}; V_{2n})$ verifying:

For any integer n≥ 2, $U_{2n}$ and $V_{2n}$ are prime numbers and $U_{2n} + V_{2n} = 2n$ .

§ 3 Introduction.

Let $(W_{2n})$ be the sequence of prime numbers defined by :
For any integer n ≥ 3,

(1) $$W_{2n} = \mathrm{Sup}(p \in IP : \ p \le 2n-3)$$

Using case disjunction reasoning, we construct two recurrent sequences of prime numbers $(V_{2n})$ and $(U_{2n})$ as a function of the sequence $(W_{2n})$ by the following process.

(2) $(V_4=2; U_4=2)$

Let n be an integer : (n≥ 3).

1) Either,

(2n – $W_{2n}$ ) is a prime number and $V_{2n}$ and $U_{2n}$ are defined directly in terms of $W_{2n}$ .

2) Either,

(2n – $W_{2n}$ ) is a composite number and $V_{2n}$ and $U_{2n}$ are defined from previous terms of the sequence $(G_{2n})$.

§ 4 Theorem.

4.1 There exists a recurrent numerical Goldbach sequence $(G_{2n}) = (U_{2n}, V_{2n})$ such that for any integer n≥ 2, we have : $U_{2n}$ and $V_{2n}$ are prime numbers and their sum is 2n.

(3) $( U_{2n}, V_{2n} \epsilon \text{ IP and } U_{2n} + V_{2n} = 2n )$

4.2 An algorithm makes it possible to explicitly calculate any term $U_{2n}$ and $V_{2n}$ .

§ 5 Methodology.

A method similar to that of the sieve is used, (see Selberg [14]). For this purpose, we build a positive sequence of prime numbers $(V_{2n})$ satisfying: lim $V_{2n} = +\infty$ , as a function of $W_{2n}$ = Sup(p ∈ IP : p ≤ 2n-3), (( $W_{2n}$) is an increasing sequence who contains all prime numbers except two, and verifies: lim $W_{2n} = +\infty$), and a complementary sequence of prime numbers $(U_{2n})$ negligible with respect to (2n) for very large integers n, i.e for any large enough integer n , we have :

(4) Sup($U_{2k}$ : (k∈IN : 3≤k ≤ n)) ≤ K1(n).log(2n).

(K1(n)>0; K1(n) quasi-constant and hyper slowly increasing function of n).

For any integer n≥ 3,

If (2n-$W_{2n}$) is a prime number “particular case”, then we put:

(5) $V_{2n} = W_{2n}$ and $U_{2n} = 2n - W_{2n}$

Else, if (2n - $W_{2n}$) is a composite number “general case” ,

we return to the sequence $(G_{2n})$ of previous terms, already calculated. Therefore, we are looking for an integer k in order to obtain two terms $U_{2(n-k)}$ and $V_{2(n-k)}$ satisfying the following conditions :

(6) $U_{2(n-k)}$, $V_{2(n-k)}$ and $U_{2(n-k)}$ +2k are prime numbers

$U_{2(n-k)} + V_{2(n=k)} = 2(n-k)$

(which is always possible ; see proof in paragraph § 6). Thus , by putting :

(7) $V_{2n} = V_{2(n-k)}$ *and* $U_{2n} = U_{2(n-k)}+2k$

we obtain two new prime numbers satisfying :

(8) $U_{2n} + V_{2n} = 2n$ .

We can thus reiterate this process by incrementing n by one unit : (n → n+1).

§ 6 Proof.
For any integer q such that : ($1 \le q \le n-3$), we have : $3 \le U_{2(n-q)} \le n$ .
For any integer k such that ($2 \le 2k \le (n-1)/2$), there exists two prime numbers $p_m$ and $p_r$, (m>r) in the interval [2;n] such that :

(9) $$p_m - p_r = 2k$$

(see Bombieri [1], Cramer [3], Iwaniec & Pintz [9] and Tchebychev [17]).
Then, there exists an integer k , ($2 \le 2k \le n-3$) such that :

(10) $$R_{2n} = U_{2(n-k)} + 2k \quad \text{is a prime number}$$

We choose the smallest integer k denoted by $k_n$ such that $R_{2n}$ is a prime number. Then we put :

(11) $$U_{2n} = U_{2(n-k_n)} + 2k_n \text{ and } V_{2n} = V_{2(n-k_n)}$$

( These two terms are prime numbers )

We have constructed in the previous steps two prime numbers , $U_{2(n-k_n)}$ and $V_{2(n-k_n)}$ whose sum is equal to $2(n-k_n)$ :

(12) $$U_{2(n-k_n)} + V_{2(n-k_n)} = 2(n-k_n)$$

Thus , by adding the term $k_n$ on each member of the equality (12), we obtain :

(13) $$U_{2(n-k_n)} + 2k_n + V_{2(n-k_n)} = 2(n-k_n) + 2k_n$$

(14) $\Leftrightarrow$ $$\{ U_{2(n-k_n)} + 2k_n\} + V_{2(n-k_n)} = 2n$$

(15) $\Leftrightarrow$ $$U_{2n} + V_{2n} = 2n$$

Finally, this algorithm constructs for any integer $n \ge 3$ two sequences of prime numbers ($U_{2n}$) and ($V_{2n}$) verifying Goldbach's conjecture .

§ 7 Corollary.
For any integer $n \ge 3$ , we easily verify :

7.1 ($W_{2n}$) is a positive increasing sequence of prime numbers.
7.2 $\lim W_{2n} = +oo$
7.3 $n < W_{2n} \le 2n-3$
7.4 $3 \le 2n - W_{2n} \le U_{2n} \le n$
7.5 $n \le V_{2n} \le W_{2n}$
7.6 $\lim V_{2n} = +\infty$
7.7 $\{ W_{2n} : n \in IN^* \} \cup \{2\} = IP$

§ 8 Remarks.

8.1 There is an infinity of integers n such that $U_{2n}$ = 3 , 5 , 7 or 11.
8.2 $V_{2n} \sim$ (2n), for (n → +oo).
8.3 For any integer n large enough, $U_{2n} \ll V_{2n}$ and $\lim(\frac{U_{2n}}{V_{2n}})$=0.
8.4 Sup($U_{2k}$ : (2≤ $k \leq n$)) ≤ K1(n).log(2n) : (K1(n)>0; K1(n) ~ $cte$).
8.5 The first integer n such that : $U_{2n} \neq$ 2n -$W_{2n}$ is obtained for n=49 and $G_{98}$= (79;19).
(This type of term "densifies" in the Goldbach sequence ($G_{2n}$) when n increases, in the sense Of Schnirelmann [13] and there is an infinity of them; we can calculate their proportion by interval).
8.6 Let q≥ 5 be an odd integer, then we could generalize this process with sequences ($W'_{2n}$) = Sup(p∈IP : p ≤ 2n-q) and we obtain other sequences ($G'_{2n}$) of Goldbach sequences independent of ($G_{2n}$).
8.7 The sequence ($G_{2n}$) is extremal in the sense that for any given integer n≥ 2 , $V_{2n}$ and $U_{2n}$are the greatest and the smallest possible prime numbers such that: $U_{2n}$ + $V_{2n}$ = 2n.

§ 9 Algorithm.
9.1 Algorithm written in natural language.
Input three variables integers: N, n and P.
Input : $p_1$=2 , $p_2$=3 , p3=5 , $p_4$ = 7 , ................, $p_N$ , the first N prime numbers
: n=3.
: P=M, R, G, S, or T given in paragraph § 2.

Start :
Algorithm body :
A) Calculate : $W_{2n}$ = Sup(p∈ IP : p ≤ 2n-3)

If $T_{2n}$= (2n – $W_{2n}$) is a prime number, then we put:

(16) $U_{2n} = T_{2n}$ and $V_{2n} = W_{2n}$

else ,
B) If $T_{2n}$ is a composite number, then let k be an integer ; we put : k=1.
B.1) while : $U_{2(n-k)}$ + 2k is a composite number,
assign to k the value k+1
return to B1)
end while .
We put : $k_n$= k and

(17) $U_{2n} = U_{2(n-k_n)} + 2k_n$ and $V_{2n} = V_{2(n-k_n)}$

Assign to n the value n+1 , (n→ $n+1$ and returh to A)
End
Output for integers less than $10^4$: Print ( 2n = ... ; 2n-3 = ... ; $W_{2n}$ = ... ; $T_{2n}$ = ... ; $V_{2n}$ = ... ; $U_{2n}$ = ...).
Output for large integers: Print ( 2n-P = ... ; 2n-3-P= ... ; $W_{2n}$-P = ... ; $T_{2n}$ = .... ; $V_{2n} - P = \cdots$; $U_{2n}$ = ... ).

9.2 Algorithm written with the language of the Maxima scientific computing software.

```
r:0 ; n1:10**500 ; for n:5*10**499+10000  thru 5*10**499+10010  do
( k:1 , a:2*n , c:a-3 , test:0 , b:prev_prime(a-1) ,
if  primep(a-b)
```

```
then print(a-n1,c-n1,b-n1,a-b,b-n1,a-b)
else ( r:r+1 ,
 while test=0 do
 ( if ( primep(c) and primep(a-c) )
then ( test:1 , print(a-n1,a-n1-3,b-n1,a-b,c-n1,a-c," Ret ",r))
else ( test:0 , c:c-2*k ))) );
```

§ 10 Appendices .

Application of the algorithm (paragraph § 9) : Table of Goldbach's sequences $(U_{2n})$ and $(V_{2n})$ calculated by the Maxima program written in paragraph § 9.2.

The abbreviation Ret) listed in the Table below indicates the results given by the algorithm in the case B) of return to the previous terms of the sequence $(G_{2n})$. WATCH OUT ! For large integers n, (2n>$10^9$, for instance), in order to simplify the writing of big numbers, we write the results in the form: 2n-P , (2n-3)-P , $W_{2n}$-P , $T_{2n}$ , $V_{2n}$-P and $U_{2n}$ with P=M, R, G, S, or T given in paragraph § 2.

| 2n I 2n-3 | $W_{2n}$ | $T_{2n}$=2n−$W_{2n}$ | $V_{2n}$ | $U_{2n}$ |
|---|---|---|---|---|
| | | | | |
| 4 1 | | | 2 | 2 |
| 6 3 | 3 | 3 | 3 | 3 |
| 8 5 | 5 | 3 | 5 | 3 |
| 10 7 | 7 | 3 | 7 | 3 |
| 12 9 | 7 | 5 | 7 | 5 |
| 14 11 | 11 | 3 | 11 | 3 |
| 16 13 | 13 | 3 | 13 | 3 |
| 18 15 | 13 | 5 | 13 | 5 |
| 20 17 | 17 | 3 | 17 | 3 |
| 22 19 | 19 | 3 | 19 | 3 |
| 24 21 | 19 | 5 | 19 | 5 |
| 26 23 | 23 | 3 | 23 | 3 |
| 28 25 | 23 | 5 | 23 | 5 |
| 30 27 | 23 | 7 | 23 | 7 |
| 32 29 | 29 | 3 | 29 | 3 |
| 34 31 | 31 | 3 | 31 | 3 |
| 36 33 | 31 | 5 | 31 | 5 |
| 38 35 | 31 | 7 | 31 | 7 |
| 40 37 | 37 | 3 | 37 | 3 |
| | | | | |
| And so on .................... | .................................... | .................................... | .................................... | .................................... |
| | | | | |

| | | | | | |
|---|---|---|---|---|---|
| 70 67 | 67 | 3 | 67 | 3 |
| 72 69 | 67 | 5 | 67 | 5 |
| 74 71 | 71 | 3 | 71 | 3 |
| 76 73 | 73 | 3 | 73 | 3 |
| 78 75 | 73 | 5 | 73 | 5 |
| 80 77 | 73 | 7 | 73 | 7 |
| 82 79 | 79 | 3 | 79 | 3 |
| 84 81 | 79 | 5 | 79 | 5 |
| 86 83 | 83 | 3 | 83 | 3 |
| 88 85 | 83 | 5 | 83 | 5 |
| 90 87 | 83 | 7 | 83 | 7 |
| 92 89 | 89 | 3 | 89 | 3 |
| 94 91 | 89 | 5 | 89 | 5 |
| 96 93 | 89 | 7 | 89 | 7 |
| R1) | | | | |
| 98 95 | 89 | 9 | 79 | 19 |
| 100 97 | 97 | 3 | 97 | 3 |
| | | | | |
| And so on.......... ......... | ..................................... | ..................................... | ..................................... | ..................................... |
| | | | | |

| | | | | |
|---|---|---|---|---|
| 120 117 | 113 | 7 | 113 | 7 |
| R2) | | | | |
| 122 119 | 113 | 9 | 109 | 13 |
| 124 121 | 113 | 11 | 113 | 11 |
| 126 123 | 113 | 13 | 113 | 13 |
| R3) | | | | |
| 128 125 | 113 | 15 | 109 | 19 |
| 130 127 | 127 | 3 | 127 | 3 |
| 132 129 | 127 | 5 | 127 | 5 |
| 134 131 | 131 | 3 | 131 | 3 |
| 136 133 | 131 | 5 | 131 | 5 |
| 138 135 | 131 | 7 | 131 | 7 |
| 140 137 | 137 | 3 | 137 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| 142 139 | 139 | 3 | 139 | 3 |
| 144 141 | 139 | 5 | 139 | 5 |
| 146 143 | 139 | 7 | 139 | 7 |
| R4) | | | | |
| 148 145 | 139 | 9 | 137 | 11 |
| 150 147 | 139 | 11 | 139 | 11 |
| | | | | |
| And so on..................... | ..................................... | ..................................... | ..................................... | ..................................... |
| | | | | |
| 180 177 | 173 | 7 | 173 | 7 |
| 182 179 | 179 | 3 | 179 | 3 |
| 184 181 | 181 | 3 | 181 | 3 |
| 186 181 | 181 | 5 | 181 | 5 |
| 188 181 | 181 | 7 | 181 | 7 |
| Ret) | | | | |
| 190 187 | 181 | 9 | 179 | 11 |
| 192 189 | 181 | 11 | 181 | 11 |
| 194 191 | 191 | 3 | 191 | 3 |
| 196 193 | 193 | 3 | 193 | 3 |
| 198 195 | 193 | 5 | 193 | 5 |
| 200 197 | 197 | 3 | 197 | 3 |
| | | | | |
| And so on..................... | ..................................... | ..................................... | ..................................... | ..................................... |
| | | | | |
| Ret) | | | | |
| 500 497 | 491 | 9 | 487 | 13 |
| 502 499 | 499 | 3 | 499 | 3 |
| 504 501 | 499 | 5 | 499 | 5 |
| 506 503 | 503 | 3 | 503 | 3 |
| 508 505 | 503 | 5 | 503 | 5 |
| 510 507 | 503 | 7 | 503 | 7 |
| | | | | |
| And so on..................... | ..................................... | ..................................... | ..................................... | ..................................... |
| | | | | |
| 1000 997 | 997 | 3 | 997 | 3 |
| 1002 999 | 997 | 5 | 997 | 5 |
| 1004 1001 | 997 | 7 | 997 | 7 |
| Ret) | | | | |

| | | | | |
|---|---|---|---|---|
| 1006 1003 | 997 | 9 | 983 | 23 |
| 1008 1005 | 997 | 11 | 997 | 11 |
| 1010 1007 | 997 | 13 | 997 | 13 |
| 1012 1009 | 1009 | 3 | 1009 | 3 |
| 1014 1011 | 1009 | 5 | 1009 | 5 |
| 1016 1013 | 1013 | 3 | 1013 | 3 |
| 1018 1015 | 1013 | 5 | 1013 | 5 |
| 1020 1017 | 1013 | 7 | 1013 | 7 |
| | | | | |
| And so on...................... | .................................... | .................................... | .................................... | ........................................ |
| | | | | |
| 5000 4997 | 4993 | 7 | 4993 | 7 |
| 5002 4999 | 4999 | 3 | 4999 | 3 |
| 5004 5001 | 4999 | 5 | 4999 | 5 |
| 5006 5003 | 5003 | 3 | 5003 | 3 |
| 5008 5005 | 5003 | 5 | 5003 | 5 |
| 5010 5007 | 5003 | 7 | 5003 | 7 |
| 5012 5009 | 5009 | 3 | 5009 | 3 |
| 5014 5011 | 5011 | 3 | 5011 | 3 |
| 5016 5013 | 5011 | 5 | 5011 | 5 |
| 5018 5015 | 5011 | 7 | 5011 | 7 |
| Ret) | | | | |
| 5020 5017 | 5011 | 9 | 5009 | 11 |
| | | | | |
| And so on..................... | .................................... | ..................................... | .................................... | ....................................... |
| | | | | |
| Ret) | | | | |
| 10000 9997 | 9973 | 27 | 9941 | 59 |
| 10002 9999 | 9973 | 29 | 9973 | 29 |
| 10004 10001 | 9973 | 31 | 9973 | 31 |
| Ret) | | | | |
| 10006 10003 | 9973 | 33 | 9923 | 83 |
| Ret) | | | | |
| 10008 10005 | 9973 | 35 | 9967 | 41 |
| 10010 10007 | 10007 | 3 | 10007 | 3 |
| 10012 10009 | 10009 | 3 | 10009 | 3 |
| 10014 10011 | 10009 | 5 | 10009 | 5 |

| | | | | | |
|---|---|---|---|---|---|
| 10016 | 10013 | 10009 | 7 | 10009 | 7 |
| Ret) | | | | | |
| 10018 | 10015 | 10009 | 9 | 10007 | 11 |
| 10020 | 10017 | 10009 | 11 | 10009 | 11 |
| | | | | | |
| And so on ..................... | ...................................... | ...................................... | ...................................... | ...................................... | |
| | | | | | |
| | | | | | |
| 2n-M | (2n-3)-M | $W_{2n}$ - M | $T_{2n}$ | $V_{2n}$- M | $U_{2n}$ |
| | | | | | |
| +1000 | +997 | +993 | 7 | +993 | 7 |
| Rt) | | | | | |
| +1002 | +999 | +993 | 9 | +931 | 71 |
| +1004 | +1001 | +993 | 11 | +993 | 11 |
| +1006 | +1003 | +993 | 13 | +993 | 13 |
| Rt) | | | | | |
| +1008 | +1005 | +993 | 15 | +919 | 89 |
| +1010 | +1007 | +993 | 17 | +993 | 17 |
| +1012 | +1009 | +993 | 19 | +993 | 19 |
| +1014 | +1011 | +1011 | 3 | +1011 | 3 |
| +1016 | +1013 | +1011 | 5 | +1011 | 5 |
| +1018 | +1015 | +1011 | 7 | +1011 | 7 |
| Ret) | | | | | |
| +1020 | +1017 | +1011 | 9 | +931 | 89 |
| | | | | | |
| And so on ...................... | ...................................... | ...................................... | ...................................... | ...................................... | |
| | | | | | |
| | | | | | |
| 2n-R | (2n-3)-R | $W_{2n}$ - R | $T_{2n}$ | $V_{2n}$ - R | $U_{2n}$ |
| | | | | | |
| Ret) | | | | | |
| +1000 | +997 | +979 | 21 | +903 | 97 |
| +1002 | +999 | +979 | 23 | +979 | 23 |
| Ret) | | | | | |
| +1004 | +1001 | +979 | 25 | +951 | 53 |
| Ret) | | | | | |
| +1006 | +1003 | +979 | 27 | +903 | 103 |
| +1008 | +1005 | +979 | 29 | +979 | 29 |

| | | | | | |
|---|---|---|---|---|---|
| +1010 +1007 | +979 | 31 | +979 | 31 |
| Ret) | | | | |
| +1012 +1009 | +979 | 33 | +951 | 61 |
| Ret) | | | | |
| +1014 +1011 | +979 | 35 | + 781 | 233 |
| +1016 +1013 | +979 | 37 | +979 | 37 |
| Ret) | | | | |
| +1018 +1015 | +979 | 39 | +951 | 67 |
| +1020 +1017 | +1017 | 3 | +1017 | 3 |
| | | | | |
| And so on..................... | ................................... | ................................... | ................................... | ................................... |
| | | | | |
| | | | | |
| 2n-G (2n-3)-G | $W_{2n}$- G | $T_{2n}$ | $V_{2n}$ - G | $U_{2n}$ |
| | | | | |
| Ret) | | | | |
| +10000 +9997 | +9631 | 369 | +7443 | 2557 |
| Ret) | | | | |
| +10002 +9999 | +9631 | 371 | +9259 | 743 |
| +10004 +10001 | +9631 | 373 | +9631 | 373 |
| Ret) | | | | |
| +10006 +10003 | +9631 | 375 | +8583 | 1423 |
| Ret) | | | | |
| +10008 + 10005 | +9631 | 377 | +6637 | 3371 |
| +10010 +10007 | +9631 | 379 | +9631 | 379 |
| Ret) | | | | |
| +10012 +10009 | +9631 | 381 | +8583 | 1429 |
| +10014 +10011 | +9631 | 383 | +9631 | 383 |
| Ret) | | | | |
| +10016 +10013 | +9631 | 385 | +9259 | 757 |
| Ret) | | | | |
| +10018 +10015 | +9631 | 387 | +4491 | 5527 |
| +10020 +10017 | +9631 | 389 | +9631 | 389 |
| | | | | |
| And so on..................... | ................................... | ................................... | ................................... | ................................... |
| | | | | |
| | | | | |
| 2n-S (2n-3)-S | $W_{2n}$- S | $T_{2n}$ | $V_{2n}$- S | $U_{2n}$ |

| | | | | |
|---|---|---|---|---|
| | | | | |
| Ret) | | | | |
| +20000 +19997 | +18031 | 1969 | +17409 | 2591 |
| Ret) | | | | |
| +20002 +19999 | +18031 | 1971 | + 17409 | 2593 |
| | | | | |
| +20004 +20001 | +18031 | 1973 | +18031 | 1973 |
| Ret) | | | | |
| +20006 +20003 | +18031 | 1975 | +16663 | 3343 |
| Ret) | | | | |
| +20008 +20005 | +18031 | 1977 | +16941 | 3067 |
| +20010 +20007 | +18031 | 1979 | +18031 | 1979 |
| Ret) | | | | |
| +20012 +20009 | +18031 | 1981 | +5671 | 14341 |
| Ret) | | | | |
| +20014 +20011 | +18031 | 1983 | +4101 | 15913 |
| Ret) | | | | |
| +20016 +20013 | +18031 | 1985 | +3229 | 16787 |
| +20018 +20015 | +18031 | 1987 | +18031 | 1987 |
| Ret) | | | | |
| +20020 +20017 | +18031 | 1989 | +16941 | 3079 |
| | | | | |
| And so on...................... | .................................... | .................................... | .................................... | ........................................ |
| | | | | |
| | | | | |
| 2n-T (2n-3)-T | $W_{2n}$-T | $T_{2n}$ | $V_{2n} - T$ | $U_{2n}$ |
| | | | | |
| Ret) | | | | |
| +40000 +39997 | +29737 | 10263 | +21567 | 18433 |
| Ret) | | | | |
| +40002 +39999 | +29737 | 10265 | +22273 | 17729 |
| +40004 +40001 | +29737 | 10267 | +29737 | 10267 |
| Ret) | | | | |
| +40006 +40003 | +29737 | 10269 | +21567 | 18439 |
| +40008 +40005 | +29737 | 10271 | +29737 | 10271 |
| +40010 +40007 | +29737 | 10273 | +29737 | 10273 |
| Ret) | | | | |
| +40012 +40009 | +29737 | 10275 | +10401 | 29611 |

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|---|---|---|---|---|
| Ret) | | | | |
| +40014 +40011 | +29737 | 10277 | -56003 | 96017 |
| Ret) | | | | |
| +40016 +40013 | +29737 | 10279 | +27057 | 12959 |
| Ret) | | | | |
| +40018 +40015 | +29737 | 10281 | +25947 | 14071 |
| Ret) | | | | |
| +40020 +40017 | +29737 | 10283 | +24493 | 15527 |

§ 11 Perspectives and generalizations.

11.1 We can consider studying other Goldbach sequences $(G'_{2n})$ and $(G"_{2n})$ independent of $(G_{2n})$ using the increasing sequences of prime numbers $(W'_{2n})$, *(see paragraph* § 8) and $(W"_{2n})$ defined by :

For any integer n ≥ 3, $W"_{2n}$ = Sup( $p\epsilon IP: p \leq$ f(n)), f being a function defined on the interval I=[3;+∞[ and satisfying the following conditions:

f is strictly increasing on the interval I, $\lim_{x\to+\infty} f(x) = +\infty$ , f(3)=3 and $\forall x\epsilon I, f(x) \leq 2x-3$.

Thus, we can choose for example, one of the following functions defined on I.

a) f: x→ax+3-3a ; (a∈ $IR$ : 0<a≤ 2).

b) g: x→ $[2\sqrt{x} - 2.\sqrt{6} + 3]$ , ( [ . ] designating the integer part of the real number x ) .

c) h : x→ $ln\,(x)$+3-ln(3).

11.2 Using this method, it would be interesting to study the Schnirelmann density [13] of certain prime numbers like 3 , 5 , 7, 11 , …….. in the sequence $(U_{2n})$ and in arithmetic sequences (Dirichlet's theorem) for n $\epsilon$ $[K_N\,;\,P_N]$ as a function of N .

11.3 It is possible to exceed the values indicated in the Table ($2n=10^{1000}$) by optimizing this algorithm, *by using supercomputers and more efficient scientific calculation software as* Maple or PARI/GP .

11.4 Diophantine equations and conjectures of the same nature (Lagrange-Lemoine-Levy's conjecture, for example) can be addressed with similar processes and algorithms . To validate the Lagrange conjecture, we can study the following sequences of prime numbers $(WL_{2n})$, $(VL_{2n})$ *and* $(UL_{2n})$ *defined by: For any integer* $n \geq 3$, $WL_{2n}$= Sup($p\epsilon IP: p \leq n - 1$) *and,*

1) If $TL_{2n}$= (2n+1 – 2.$WL_{2n}$) is a prime number, then we put: $VL_{2n}= WL_{2n}$ and $UL_{2n}= TL_{2n}$.

2) If $TL_{2n}$ is a composite number, then there exists an integer k , (1≤ $k \leq n-3$) such that: $UL_{2(n-k)}$ + 2k is a prime number and we put: $VL_{2n}$=$VL_{2(n-k)}$ and $UL_{2n}$=$UL_{2(n-k)}$+ 2k .

3) With the same type of process, one can validate generalized Bezout-Goldbach's conjectures of the form:

a) Given two integers K and Q odd and coprime, for any integer n such that : (2n ≥ 3(K+Q)), there exist two prime numbers $U'''_{2n}$ and $V'''_{2n}$ verifying : K.$U'''_{2n}$ + Q.$V'''_{2n}$ = 2n.

b) Given two integers K and Q of different parity and coprime, for any integer n such that : (2n≥ 3 (K+Q)), there exist two prime numbers $U'''_{2n}$ and $V'''_{2n}$ verifying : K.$U'''_{2n}$ + Q.$V'''_{2n}$ = 2n + 1.

§ 12 Conclusion.

12.1 We have explicitly constructed a unique Goldbach sequence $(G_{2n}) = (U_{2n};V_{2n})$ such that $U_{2n}$ and $V_{2n}$ are prime numbers satisfying : $U_{2n}$+$V_{2n}$= 2n for any integer n≥ 2 , using a simple and very efficient unconventional "local recursive" algorithm.

12.2 We broke the record held by Silva [15] on a personal computer, and we were able to reach values of the order of $2n=10^{1000}$ with a reasonable calculation time (less than three hours for the evaluation of ten values of $U_{2n}$ and $V_{2n}$) .

12.3 For a given integer n, the evaluation of the terms $U_{2n}$ and $V_{2n}$ does not require the calculation of all the previous terms $U_{2k}$ and $V_{2k}$ , (1≤k<n-1). You just need to know the prime numbers $p_l$ and the terms $V_{2r}$ in a neighborhood of 2n , (on the interval J = [2n-K1.log(2n);2n]). This property allows a fast calculation of $U_{2n}$ and $V_{2n}$ even for values of 2n of the order of $10^{1000}$ .

12.4 Therefore, the strong EULER-GOLDBACH conjecture is true.


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Adress : PHILIPPE SAINTY , 9 Allée Robert de Wey , 93190 LIVRY-GARGAN,. (FRANCE).

Email : psainty.math@gmail.com