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**Abstract**

Electrophysiological brain source localization consists in estimating the source positions and activities responsible for the (S)EEG measurements. The localization procedure is usually carried out in the time domain, however in specific situations the activities of interest can be located at well defined frequencies, e.g. in response to a rhythmic stimulation. This paper addresses the problem of sparse localization of multiple sources oscillating at the same frequency. In particular the non-unicity of the solution is emphasized, as alternative source maps involving equivalent or less number of sources can be found, challenging source localization methods based on sparsity. These limitations are illustrated under a realistic SEEG simulation framework, and the usefulness to perform localization for this modality is strengthen out.

**Index Terms**

EEG source localization, frequency domain, sparse inverse problem

## I. INTRODUCTION

Brain source localization consists in the estimation of the underlying generators from the electroencephalographic recordings. Since the number of recording channels is far less than the size of the source space, this source reconstruction problem is known to be severely ill-posed. Priors on the underlying sources are needed for regularization, and high signal to noise ratio are desirable to reduce estimation uncertainties.

In this paper the case of sources with activities with specific frequency components is considered, such situations being specifically met in stimulation protocols involving a stimulus (e.g. visual or auditory) at a given frequency, causing rhythmic activities of the responding brain structures at the same frequency (possibly with harmonics), and known as steady-state evoked potentials [1], [2]. It is useful to note also that, at least for certain cognitive protocols using fast periodic (visual) stimulations (FPVS), it was shown using depth recordings that rather few brain areas are responding, although their limits are not easy to identify [1]. For these protocols, the characteristics of the data suggest that localization methods should focus on sparsity and exploit the periodicity (*i.e.*, the frequency characteristics) of the recordings. Indeed, while a wide range of localization methods have been developed directly in the time domain [3], few studies have evaluated the opportunity to solve the problem in the frequency domain ( [4]–[6]). The first reason of such approaches is to enhance the signal to noise ratio by focusing on the information (frequency peaks) of interest. As we showed in [2], sparse source localization can be performed successfully in the frequency domain using greedy algorithms from the OLS family, namely the SBR [7], which outperform state of the art scanning approaches such as RAP- or TRAP-MUSIC [8], [9] applied in time domain, even after averaging over the different periods of the signal (see [2] for details).

The objective of the paper is to analyse further the source localization problem of sparse oscillatory sources having the same frequency but being possibly shifted in phase. We give theoretical evidence that alternative solutions exist and are likely to mislead the localization results, *i.e.*, the projection of an arbitrary number of oscillatory sources on the sensors can be equivalently reproduced by the sum of alternative oscillating sources at the same frequency, provided that appropriate projection patterns for these subsidiary sources can be found in the considered lead-field.

We illustrate this phenomenon on a SEEG setup, which may be particularly vulnerable due to the higher degree of freedom offered by the lead-field where the orientations are usually left unconstrained, and due to the particular geometry induced by the SEEG electrode implantation leading to a poorly conditioned problem [10]. We take the opportunity of this study to emphasize once more the necessity of performing source localization from this modality [10], [11], but from a different point of view in the Fourier domain.

## II. METHOD

### A. Model

Our previous work [2] focused on the simplest model, corresponding to pure oscillating sources (sinusoidal signals having the same frequency but potentially different phases and amplitudes) projected to the sensors via a known fixed orientation lead-field matrix. The model writes:

$$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{s}_i a_i \sin(\omega_0 t + \varphi_i) \tag{1}$$

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where $\mathbf{x}(t) \in \mathbb{R}^{M\times 1}$ is a column vector representing the $M$ electrodes measurements at instant $t$, $\mathbf{s}_i$, $i = 1 \dots N$ are the $N$ fixed orientation lead-field columns ($M \times 1$) encoding the positions and the orientation of $N$ sources, $a_i$ and $\varphi_i$ being the amplitudes and phases of those sinusoidal sources, all at $f_0 = \omega_0/2\pi$ frequency.

Our goal is to explore further the localization problem for periodic sources. In particular, we aim to relax the fixed AND known orientation constraint: we consider thus a model in which the sources have a fixed orientation, BUT unknown. In other words, model (1) writes

$$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{K}_i \mathbf{j}_i a_i \sin(\omega_0 t + \varphi_i) \tag{2}$$

where $\mathbf{K}_i$ is now $M \times 3$ and encodes the full 3-dimensional lead-field for a given position $i$ and $\mathbf{j}_i$ ($3 \times 1$) is the orientation of the dipolar source at that specific position.

From a source localization point of view, model (1) implies estimating the position, the amplitude and the phase of the source, while model (2) implies estimating also its orientation. The usefulness of this model can be debated in EEG based localization problems, where one can make the hypothesis that the sources visible in EEG are mainly on the cortical surface and oriented orthogonal to the head surface [12]. However this is particularly relevant when dealing with SEEG recordings, where sensors are implanted in the brain and capture sources having orientations determined by the local (folded) cortex.

### *B. Ambiguity problem*

The question of the conditioning is addressed in this section: since we have relaxed the orientation constraint, does the localization problem still accept a unique solution under sparse approximation.

Assume that the $N$ sources have fixed unknown orientations. In this case, model (1) holds with unknown $\mathbf{s}_i$ gains obtained as linear combinations of the 3 columns of $\mathbf{K}_i$ (2). The model can be further developed[1] as:

$$\begin{aligned}
\mathbf{x}(t) &= \sum_{i=1}^{N} \mathbf{s}_i a_i \sin(\omega_0 t + \varphi_i) \\
&= \sum_{i=1}^{N} \left(\mathbf{s}_i a_i \cos(\varphi_i)\sin(\omega_0 t) + \mathbf{s}_i a_i \sin(\varphi_i)\cos(\omega_0 t)\right) \\
&= \left(\sum_{i=1}^{N} \mathbf{s}_i a_i \cos(\varphi_i)\right)\sin(\omega_0 t) + \\
&\qquad \left(\sum_{i=1}^{N} \mathbf{s}_i a_i \sin(\varphi_i)\right)\cos(\omega_0 t) \\
&= \mathbf{c}_s \sin(\omega_0 t) + \mathbf{c}_c \cos(\omega_0 t)
\end{aligned} \tag{3}$$

As shown above, a mixture of $N$ sources can be reduced to a mixture of 2 sources only (a sine and a cosine), with projection gains $\mathbf{c}_s$ and $\mathbf{c}_c$ ($M \times 1$). If these two column vectors are equal or close to some $\mathbf{s}_j$ and $\mathbf{s}_k$ vectors from the lead-field matrix, then we have an ambiguity problem - a greedy source localization algorithm risks to choose the two "alternative" sources solution instead of the correct $N$ sources one. For a fixed orientation lead-field as in model (1), this is rather unlikely because it is more constrained: original sources should have particular amplitudes and phases such as the linear combination of the original $\mathbf{s}_i$ columns yield some other $\mathbf{s}_k$ column (except for the case when the original sources are close between them and thus have similar $\mathbf{s}_i$ projections). But, when the sources have unknown orientations, the problem has more degrees of freedom: if the vectors $\mathbf{c}_s$ and $\mathbf{c}_c$ "almost" lie in the span of some 3-columns lead-field blocks $\mathbf{K}_j$ and $\mathbf{K}_k$, that is, if there exists a linear combination (thus an orientation $\mathbf{j}_k$) of the 3 columns of a block that approximates the "alternative" gains $\mathbf{c}_s$ and $\mathbf{c}_c$, then a source localization algorithm might converge to these alternative locations. To put it differently, if the regression of the column $\mathbf{c}_s$ on some group of three columns $\mathbf{K}_j$ of the lead-field is good enough (*i.e.*, if the projection $\hat{\mathbf{c}}_s$ of $\mathbf{c}_s$ onto the subspace spanned by $\mathbf{K}_j$ is a good approximation of $\mathbf{c}_s$), then the problem becomes harder. Indeed, if the error is small, it will be impossible to distinguish between this kind of alternative solution and the results obtained for noisy signals or for lead-field model errors. The first results presented in the next section aim to quantify the frequency of this ambiguity problem: is it a kind of degenerate case seldom encountered in real localization problems or it is much more commonly encountered?

[1]Note that this development is a particular case, one can always split $\varphi_i$ in a fixed common phase for all sources $\varphi_0$ and a variable part $\delta_{\phi,i}$, in which case the columns $\mathbf{c}_s$ and $\mathbf{c}_c$ will have different values.

## C. *Usefulness of periodic source localization in SEEG*

A second legitimate question is: is it necessary to perform source localization in SEEG or, as often considered, SEEG "sees" local activities and by itself can be considered a golden standard for source localization (EEG procedures are confronted with SEEG "ground-truth" for example in [13]). As it was shown in [10], [11], [14], source localization can be performed and it is relevant in SEEG for epileptic spikes for example. The second set of results that we will present in this paper enforce the fact that SEEG localization is a valuable pre-processing step. In principle, this should allow a better spatial resolution when determining the brain areas responsible for specific answers such as face perception (*e.g.*, using FPVS protocols [1]).

# III. RESULTS AND ANALYSIS

## A. *Simulation setup*

SEEG recordings were simulated using a realistic head model with five isotropic compartments (gray matter, white matter, cerebrospinal fluid (CSF), bone and scalp), for which we have built a Finite Element Model (see [10] for more details). The source space consisted of 509 positions regularly sampled (nodes of the tetrahedral mesh of the gray matter, approximately situated on 1cm grid). A realistic SEEG implantation with 12 electrodes, 9 in the right hemisphere and 3 in the left ($M = 186$ sensors in total) was used (see figure 2). The resulting lead-field matrix $\mathbf{K}$, $186 \times (3 \cdot 509)$, was used to generate SEEG potentials from the active sources.

Three sources were simulated ($N = 3$). The time courses of the 3 sources were sinusoidal signals at a given common frequency $f_0$ (here 6 Hz) but having different (random) amplitudes $a_i$ and phases $\varphi_i$ (amplitudes according to a normal distribution with a mean of 5 (arbitrary units) and a standard deviation of 1, phases drawn from a uniform distribution on $[0, \ \pi]$). The sampling frequency was 2100 Hz and the duration 10s.

The positions and orientations of the 3 sources were selected randomly (orientations according to a normal distribution with 0 mean and positions among the 509 possible ones).

One thousand simulations were performed varying the random parameters above, with no noise.

## B. *Ambiguity analysis*

For all the 1000 simulations, we computed the alternatives $\mathbf{c}_s$ and $\mathbf{c}_c$ according to (3). Next, we have scanned the 509 positions embedded in the lead-field matrix $\mathbf{K}$ and regressed $\mathbf{c}_s$ and $\mathbf{c}_c$ on each 3D-subspace spanned by the columns corresponding to each position, obtaining thus the orientation as the regression coefficients. The projection $\hat{\mathbf{c}}_s$ of $\mathbf{c}_s$ (same procedure for $\mathbf{c}_c$), *i.e.*, its approximation as a linear combination of the three columns of the lead-field at each position, was computed for every position. Finally, we have chosen the best among the 509 approximations to identify the alternative source position $i_s$ and its projections $\hat{\mathbf{c}}_s$:

$$i_s = \underset{i}{\operatorname{argmin}}(\|\mathbf{c}_s - \mathbf{K}_i \mathbf{K}_i^+ \mathbf{c}_s\|) \tag{4}$$

$$\hat{\mathbf{c}}_s = \mathbf{K}_{is} \mathbf{K}_{is}^+ \mathbf{c}_s \tag{5}$$

with $\mathbf{K}_i^+$ the pseudo-inverse of the $(M \times 3)$ $\mathbf{K}_i$ block.

Alternative measured signals were next computed as:

$$\hat{\mathbf{x}}(t) = \hat{\mathbf{c}}_s \sin(\omega_0 t) + \hat{\mathbf{c}}_c \cos(\omega_0 t) \tag{6}$$

Considering the whole signal length, the goodness of fit (GOF) between the original signals $\mathbf{X}$ and the ones reconstructed from the alternative sources (6) is defined as:

$$GOF = 1 - \left( \frac{\|\mathbf{X} - \hat{\mathbf{X}}\|}{\|\mathbf{X}\|} \right)^2 \tag{7}$$

To summarize, at every iteration we have generated the potentials $\mathbf{X}$ using 3 sources at random positions $i_{\{1,2,3\}}$ and we computed an alternative data matrix $\hat{\mathbf{X}}$ from only two sources, with positions given by $i_s$ and $i_c$ and orientations and amplitudes given by regression ($\mathbf{K}_{is}^+ \mathbf{c}_s$ for the sine source, $\mathbf{K}_{ic}^+ \mathbf{c}_c$ for the cosine source). We have grouped the simulation outcomes in three sets: $2/2$, when both $i_s$ and $i_c$ are within a distance of 10 mm from one of the original sources $i_{\{1,2,3\}}$, $1/2$ when only one of the two alternative sources is within this distance and $0/2$ when both alternative sources are far from the original ones.

As seen in figure 1, in about a quarter of the simulations, the two equivalent sources are close to the original ones (set $2/2$) and in three quarters at least one is far from them. Meanwhile, the GOFs for the three situations are high enough (above 0.8) and have comparable values. In other words, a source localization algorithm, especially in noisy situations, might easily converge to a false solution (an alternative one). Indeed, a 10dB signal to noise ratio for example will also yield a GOF (between the noisy signal and the original one) at about 0.87 (at 3dB, the GOF falls at 0.38).

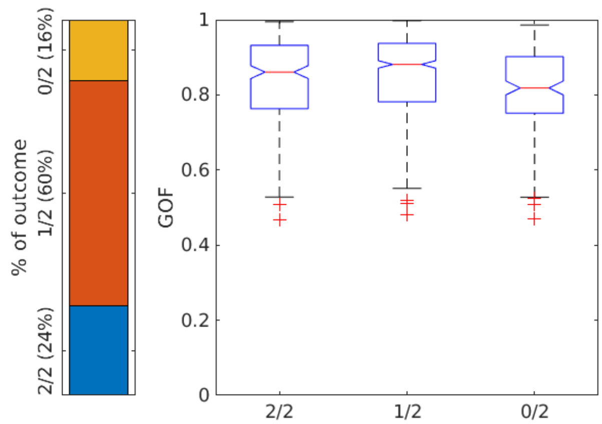


Fig. 1. Ambiguity results. Left panel, the proportion of different outcomes (in blue (24%): both alternative sources were among the 3 simulated ones; in red (60%): one of the sources was far from the simulated ones; in yellow (16%): both alternative sources were far from the simulated ones. Right panel, the GOF of the reconstructed signal $\hat{\mathbf{X}}$ with respect to the original $\mathbf{X}$ for the three possible outcomes in the left panel.

### C. *Usefulness analysis*

SEEG recordings are often considered as ground truth for source localization [1], [13]. The underlying hypothesis is that SEEG signals are dominated by sources in the immediate neighbourhood of the sensors. We take advantage in this section of the oscillatory sources setup to illustrate (using frequency domain plots) the fact that SEEG sensors record mixtures of sources coming from different positions. One example is taken among those simulated above: three sources with the same frequency but different orientations, phases and amplitudes were placed as shown in figure 2 (the case corresponds to the $0/2$ set).

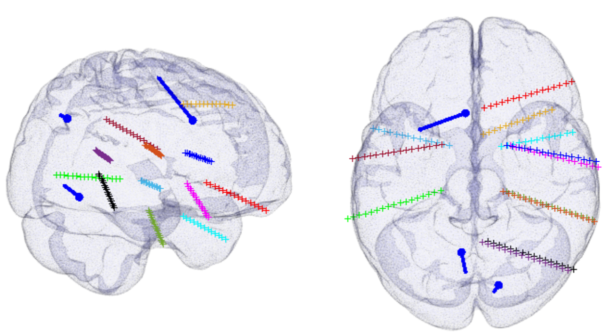

Fig. 2. Three sources placement in the brain. They are illustrated in blue arrows representing positions and orientations of sources. Electrodes are represented as crosses and each one with a different color. For illustrative purposes, source amplitudes are shown 40% larger of their original.

We computed the Fourier transform of the $M$ sensor signals and displayed their magnitudes and phases for the frequency of interest $f_0$. If the sources do not mix together, the ambiguity problem discussed above is negligible and every source should yield signals on the close sensors only, which means that the phases of these sensors should be the same (for sensors seeing a single source). In other words, in the polar plot 3 we should have three straight lines passing through the center, one for each source (*e.g.*, sensors having the same phase), with sensors close to the sources having a greater magnitude (radius). An illustrative example is the source recorded by the sensors in yellow: it yields big amplitudes, although it is situated in the opposite hemisphere; its phase alignment might indicate on the other hand that it is not disturbed by any other surrounding activity. However, most of the other sensors show mixtures of phases: the two sources in the posterior part of the brain either mix together (as it seems to be the case on the sensors of the purple electrode) or they are seen individually (black sensors). The red and the brown electrodes, although with smaller amplitudes, both seem to be mixtures of sources, which could be disentangled by performing source localization. In this case, RAP-MUSIC finds two sources (GOF = 0.97) one correct (the frontal one), and the other one in the posterior part of the brain, between the two actual ones.

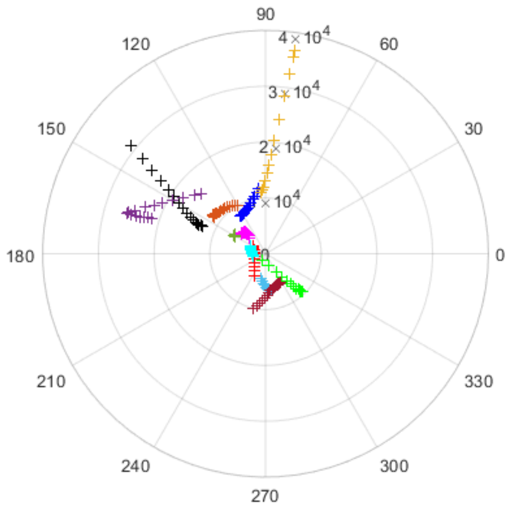


Fig. 3. Polar plot of the sources in figure 2

## IV. CONCLUSIONS

We have analyzed in this paper the problem of localizing multiple oscillatory sources at even frequency. We showed that ambiguities are likely to occur in the source map when multiple oscillatory sources with fixed but unknown orientations are projected on the sensors, as the data might be reconstructed equivalently by a set of alternative sources.

The simulations performed in this paper then indicate that the source localization problem for periodic freely oriented sources is particularly challenging. New algorithms need to be developed for this type of problem, able to tackle in the frequency domain (where the SNR is optimal) the estimation of both the position and the orientation of the sources under a sparsity constraint. Applying this type of algorithm to FPVS protocols could help defining more precisely the active areas of the brain during specific cognitive tasks and, potentially, estimating the delays among them.


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## REFERENCES


[1] J. Jonas *et al.*, "A face-selective ventral occipito-temporal map of the human brain with intracerebral potentials," *Proceedings of the National Academy of Sciences*, vol. 113, no. 28, pp. E4088–E4097, 2016.

[2] V. del Rocío Hernández-Castañón, S. Le Cam, and R. Ranta, "Sparse EEG source localization in frequency domain," in *2021 43rd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC)*. IEEE, 2021, pp. 6428–6432.

[3] C. Michel *et al.*, "EEG source imaging," *Clinical neurophysiology*, vol. 115, no. 10, pp. 2195–2222, 2004.

[4] P. A. Muñoz-Gutiérrez *et al.*, "Localization of active brain sources from EEG signals using empirical mode decomposition: A comparative study," *Frontiers in integrative neuroscience*, vol. 12, p. 55, 2018.

[5] X. Zheng and B. Wan, "Study on EEG source localization in frequency domain," in *27th IEEE EMBC Annual Conference*, 2005, pp. 5958–5961.

[6] B. Lütkenhöner, "Frequency-domain localization of intracerebral dipolar sources," *Electroencephalography and clinical Neurophysiology*, vol. 82, no. 2, pp. 112–118, 1992.

[7] C. Soussen *et al.*, "From Bernoulli–Gaussian deconvolution to sparse signal restoration," *IEEE Transactions on Signal Processing*, vol. 59, no. 10, pp. 4572–4584, 2011.

[8] J. C. Mosher and R. M. Leahy, "Source localization using recursively applied and projected (RAP) MUSIC," *IEEE Transactions on signal processing*, vol. 47, no. 2, pp. 332–340, 1999.

[9] N. Mäkelä *et al.*, "Truncated RAP-MUSIC (TRAP-MUSIC) for MEG and EEG source localization," *NeuroImage*, vol. 167, pp. 73–83, 2018.

[10] V. Caune *et al.*, "Evaluating dipolar source localization feasibility from intracerebral SEEG recordings," *NeuroImage*, vol. 98, pp. 118–133, 2014.

[11] S. Le Cam *et al.*, "SEEG dipole source localization based on an empirical bayesian approach taking into account forward model uncertainties," *NeuroImage*, vol. 153, pp. 1–15, 2017.

[12] A. M. Dale and M. I. Sereno, "Improved localization of cortical activity by combining EEG and MEG with MRI cortical surface reconstruction: a linear approach," *Journal of cognitive neuroscience*, vol. 5, no. 2, pp. 162–176, 1993.

[13] L. Koessler *et al.*, "Source localization of ictal epileptic activity investigated by high resolution EEG and validated by SEEG," *Neuroimage*, vol. 51, no. 2, pp. 642–653, 2010.

[14] S. Le Cam *et al.*, "Dealing with the SEEG sparse setup: a local dipole fitting strategy," in *2019 9th International IEEE/EMBS Conference on Neural Engineering (NER)*. IEEE, 2019, pp. 1003–1006.